

metals, when subjected to very low temperatures, which flowed from his early collaboration with Sir James Dewar in investigating this domain. In recent years he has taken a prominent part in the scientific development of telegraphy by free electric waves.

Influence of Viscosity on the Stability of the Flow of Fluids.

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No part of the theory of hydrodynamics is more difficult than that relating to stability, and even were the conditions which determine the limits of stability thoroughly and satisfactorily investigated, only the fringe of the problem would be touched, for the chief interest in the cases presented by real fluids lies in the character of the motion after instability has been established.

In some restricted dynamical problems (*e.g.* an inverted pendulum) the system passes from an unstable to a stable configuration, and the whole process can be traced. With fluid motions, however, this is not the case, and there is no more prospect of tracing the course of a particle of fluid in an unstable flow than of following an individual molecule of gas in its various encounters with its neighbours.

Nevertheless, the direction and speed in a region of unstable flow is not in general a mere matter of chance, but rather one of periodic variation, though the periods involved are not in most cases definite, in the same sense as are the periods of stable systems, but are in character more nearly allied to the intervals which separate the times of activity of a geyser—intervals, that is, whose constancy depends on the uniformity of the rate at which energy is supplied or withdrawn from the fluid.

In all the streams, whether of gas or liquid, met with in nature, the conditions are in general those of instability, and it is only in certain cases, and by taking considerable precautions, that an approximately stable flow can be maintained, and where this is accomplished the stream-lines are of the electric type.

Although it is impossible by mathematical analysis to follow in detail the motion in an unstable flow, the general character may, in many cases, be traced, and the object of the present note is to apply an observation made by

the late Mr. Froude to the explanation of some of the commonly occurring phenomena of instability.

At the meeting of the British Association at Bristol in 1875, Froude showed,* among other experiments, one in which a jet issuing horizontally from an aperture in a vessel of water impinged symmetrically on a similar aperture in a vessel standing near the first (fig. 1). The water in the

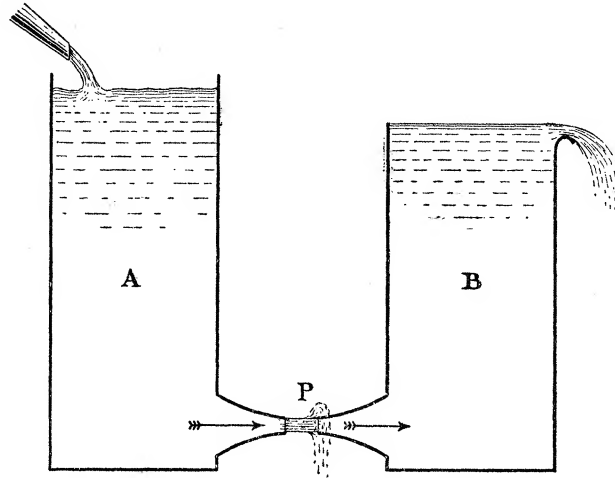


FIG. 1.

first vessel (A) was maintained at a uniform level, and by providing an outflow in the second vessel (B) at a slightly lower level, the stream from one to the other could be maintained indefinitely, nearly the whole of the outflow from A entering B at the speed due to the head. A small but definite loss of fluid, however, occurred at the entrance to the aperture in B, giving rise to the appearance shown at P in fig. 1. It will be seen that the loss is confined to a thin layer at the outer surface of the jet—to that part of the jet in fact which was in contact with or close to the walls of the passage from which it issued; and Froude pointed out that this was to be explained by the loss of velocity suffered by the outer part of the jet from surface friction and viscosity, for the fluid which has had its velocity lowered without having its pressure raised has had its potential degraded, and is therefore incapable of ever again entering a region where the pressure is greater than that due to its diminished velocity. The degradation of potential by viscous action gives the explanation of a very large number of natural phenomena, some of which are on a large scale, but before giving examples it

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will be convenient to examine the nature of viscous degradation more closely.

Suppose a stream of fluid flowing in a parallel channel with a velocity $v = \sqrt{2gH}$ (uniform over the whole section to begin with) has its velocity over a certain thickness near the walls reduced from v to θv by the action of surface friction and viscosity; the pressure is not altered in the layer, but the height to which its velocity can carry it is reduced from $\sqrt{H}$ to $\sqrt{\theta H} = \sqrt{H'}$ so that $H'/H = \theta^2$. Now suppose the channel to contract; the stream-line pressure diminishes as the velocity increases, and there is nothing to prevent the degraded surface streams from retaining their surface position. If, however, the channel expands so as to make the stream-line pressure greater than H' , the degraded streams must have their onward flow diverted or reversed, and in either case the conditions of continuity will produce a change of flow in the central streams, different, not only in degree, but in kind, from any change due to the expansion of the electrical stream-lines.

Thus it will be seen that according to whether the electrical stream-lines are convergent or divergent, the results in the presence of viscous dissipation are of widely different characters. When the electrical stream-lines are convergent, the actual path of the retarded streams may be nearly the same as if viscosity were absent; but with divergent electrical flow this is impossible, if the divergency is such as to raise the pressure anywhere above that which is due to the speed of the degraded streams.

There is a difficulty in many real phenomena in distinguishing between the secondary currents arising from the conditions of continuity due to the diversion or reversal of the degraded streams, and the degraded streams themselves. Some sort of an eddy is necessarily formed, and although it would be too much to say that all eddies in real fluids are formed in this way, the following examples in which the explanation does apply will be sufficient to show how far-reaching the effect of degraded potential is, and how large what may be called the secondary effects of viscosity are, compared with those which appear in the initial stages of instability investigated by mathematicians:—

(1) It is well known that in many tidal waters (*e.g.* the English Channel) the stream continues to run in mid-channel after it has turned inshore. In this case the shallow water near the shore is more retarded than the mid-channel surface streams, and thus since the level of the water is not affected by viscous retardation, the degraded inshore stream is unable to hold up against the tidal gradient after this has reached some definite value (fig. 2, $a-f$). The same reasoning applies to all the fluid which is near enough to the boundary of the channel (whether bed or shore) to be sensibly

influenced by surface friction, and there can be no doubt that the tidal current along the bed of the channel is also reversed before that on the surface. In many tidal rivers this is known to be the case from actual observation.

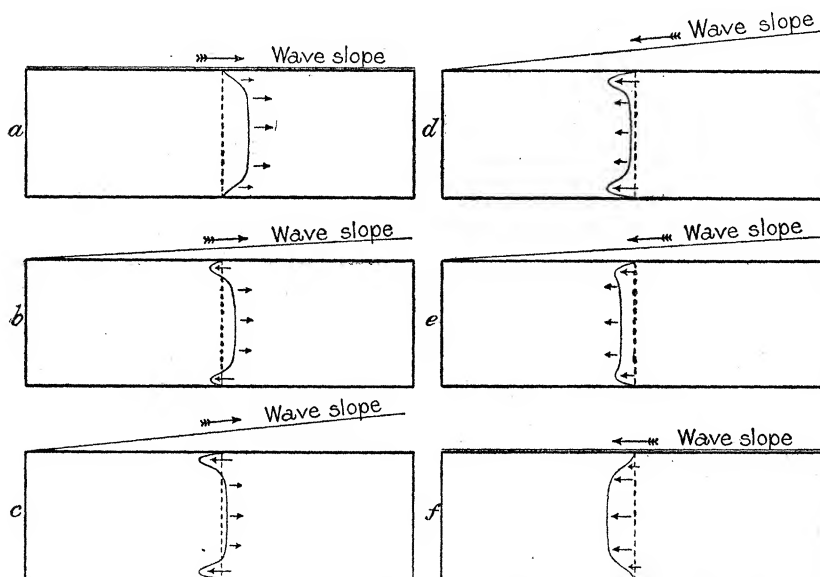


FIG. 2, *a-f*.—Diagrammatic plan of the direction of flow in a viscous fluid oscillating in a rectangular channel during one half period. *c* and *d* refer respectively to the times just before and just after the epoch of maximum wave slope.

(2) I have watched the same phenomenon in a canal under the following circumstances :—When a lock is being filled a stream is created in the canal, and as the lock fills this stream is gradually retarded and the water level at the lock rises in consequence above the mean level of the canal by an amount depending on the velocity of the stream (among other things). The stream in the canal is, of course, from the surface friction, slower at the sides than in the middle, and the side current there is reversed by the excess of head at the lock long before the central stream has ceased to flow in its primary direction. I have seen this reversal more than a quarter of a mile above a lock in a canal whose width did not exceed 30 feet.

The opposite directions of the side and central streams imply a vorticity in the fluid, and constitute together (including the reversal over the bed) a single long eddy. This is in itself unstable, and secondary eddies are formed along its length as in fig. 2, *g*. These, however, are generally difficult to observe, for the stream to begin with contains eddies of the same character, which tend to mix the layers at different distances from the

boundaries, and it is on this account the speed of rivers is more uniform over the cross-section than it would be were the flow lamellar.



FIG. 2, *g*.—Diagrammatic vertical section, showing the eddies in a current flowing in a rectangular channel.

(3) Another very typical case mentioned by Froude is the inward spiral motion which occurs near the bottom of a flat circular vessel while water rotating in it is coming to rest, which is made apparent by the way in which small objects are carried towards the centre.

Near the sides and floor the degraded streams are unable to describe plain circular orbits from lack of sufficient velocity to withstand the pressure due to the motion of the less retarded fluid, the result being that the centre of curvature of the orbit is shifted and the stream deflected inwards. The inward stream over the floor necessitates an upward central current and a slow outward flow at a higher level giving rise to a circulation such as is shown in fig. 3*a*.

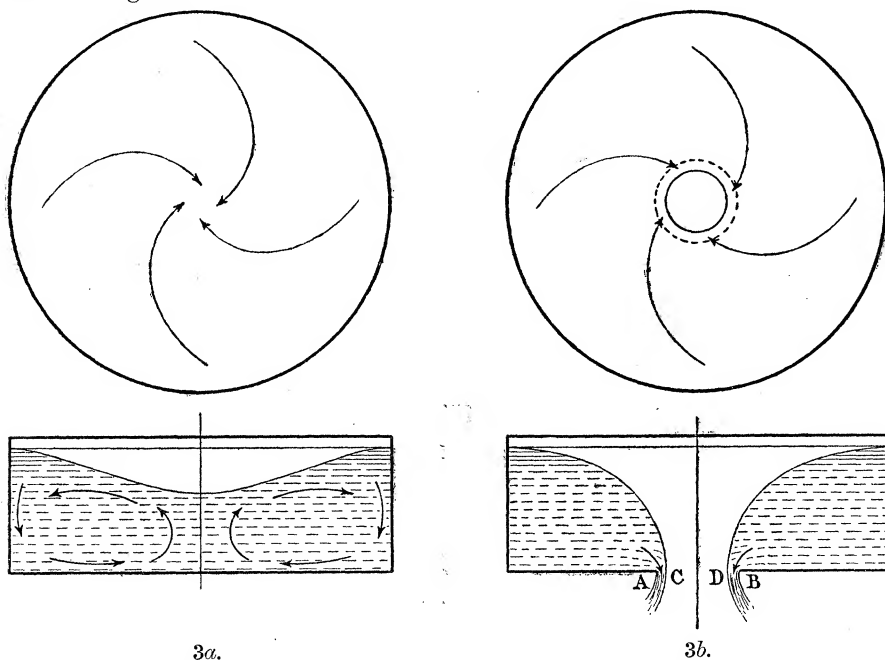


FIG. 3*a*.—Diagrams showing in plan the spiral current near the floor, and in section the vertical circulation set up when a viscous fluid which has been rotating in a circular vessel is being brought to rest by the action of the stationary boundaries.

FIG. 3*b*.—Plan and section of a vortex in a viscous fluid flowing from a central hole in a circular vessel.

The same phenomenon has been treated by Prof. James Thomson ('Roy. Soc. Proc.' 1876), who explained in this way the formation of banks of gravel and pebbles which are found on the inside of the bends of quickly flowing rivers. In the slower tidal waters of estuaries the curious conformation of sand banks is a result of the same kind of action.

(4) As another example suppose fluid to be flowing from a circular hole in the floor of a large vessel in which the level at a distance from the outlet is maintained constant. If the fluid is without rotation the flow to begin with is purely radial. A flow of this character is unstable, and even the smallest angular velocity given to the mass will increase as the flow proceeds and ultimately the fluid itself will form a vortex, at the free surface of which the orbital velocity is that due to the difference of level between the point considered and the surface at a distance (fig. 3*b*). The effective area of the outlet is then determined by the difference of the diameters of the outlet and the tubular vortex surface at the floor level (viz., $AB-CD$ in the figure). If the liquid is devoid of viscosity $AB-CD = 0$ and the outflow ultimately ceases, but with a viscous fluid $AB-CD$ is always finite and the ultimate condition is reached when the energy in the fluid discharged is equal to the work expended in viscous dissipation within the vessel.

(5) Cases analogous to (3) and (4) occur in meteorology. The ordinary barometric depressions of large area may be considered as vortices undergoing viscous dissipation whose only store of energy is the kinetic energy due to velocity. These correspond to the circulation of liquid in a closed vessel as in example (3). The loss of velocity near the ground necessarily causes the degraded streams to turn inwards towards the centre, and the conditions of continuity demand that near the centre there must be an upward current.

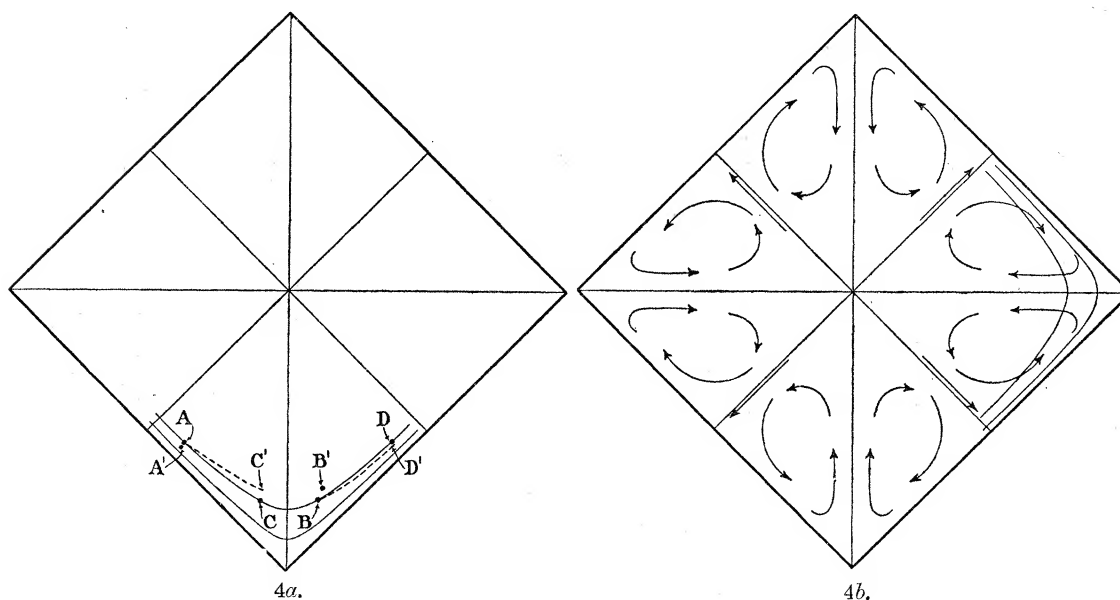
Whirlwinds, waterspouts, dust storms, and perhaps tornadoes have part of their energy in the potential form due to denser air lying for a time unstably over air which is lighter, and which when disturbed rushes up a chimney as it were of its own formation. These are analogous to (4). In both cases the inward slope of the spirals followed by the current depends on the amount of degradation which the stream has undergone; but for ordinary meteorological depressions the upward central current is a secondary effect of the degraded circulation, whilst in whirlwinds, etc., it is the primary cause.

(6) Similar considerations apply to slowly oscillating liquid. The simplest case is that in which the fluid completely fills a long prismatic box oscillating about an axis parallel to the edges of the prism.

The motion of the fluid far removed from the ends is then two-dimensional. The electric stream lines of the fluid in these circumstances are known, and it is not necessary for the present purpose to define them by symbols.

The relative velocity of the fluid and the solid boundary is a maximum through the planes parallel to the axis of rotation and passing normally through the middle of each side (if the prism is a regular polygon) and a minimum through the planes bisecting the angles. The general character of the flow is shown by the dotted lines in fig. 4*a*. In a perfect fluid the velocity and pressure are of course connected in the usual way.

If the fluid is viscous, consider the motion of two elements which start from A and B respectively and which in the absence of viscosity would follow the paths AC and BD. Here A moves from a region of low towards a region of higher pressure, while B travels from a higher towards a lower pressure.



FIGS. 4*a* and 4*b*.—Slow circulation set up in a viscous fluid completely filling a rectangular prismatic box oscillating about the axis of the prism. In Fig. 4*a*, A C B D is a stream line of perfect fluid referred to the boundary considered as stationary.

Owing to viscous retardation the paths AC' and BD' will both be shorter than AC and BD, but while there is no reason (except such as may be imposed by the conditions of continuity) that D' should not lie on BD, it is impossible that C' should lie on AC, for at all points along this path the pressure is greater than the head equivalent to the degraded velocity of the particle which starts from A. The actual path AC' must therefore lie inside and be more curved than AC. In the next quarter of an oscillation C' will return not to A but to A' outside A and D' to B' inside B.

The result of the first half of an oscillation is that a certain amount of

fluid will be carried inwards from the angles towards the centre of the prism and outwards from the centre towards the middle of the sides. The same process will go on in the succeeding half oscillation and if continued will give rise to a slow circulation such as is shown by the arrows in fig. 4*b*.

(7) An important case is that of the initiation of eddies in a stream flowing past an obstacle.

Let the obstacle be a lamina. As is well known there are two forms of mathematical solutions of this problem, one of which makes the stream-lines similar curves both up and down stream (electric flow) while the other involves a surface of discontinuity on the down stream side. As far as this portion of the flow is concerned, neither of the solutions represents even approximately what is observed to take place in real fluids, though the discontinuous form is not so far removed from the facts as is the electric flow. The form of the discontinuous surface of theory is determined by making its curvature such that, while fulfilling the conditions of continuity, the pressure just outside the surface and throughout the space within it is that of the fluid at a distance, the velocity just outside the boundary being that of the general stream and zero inside, fig. 5.

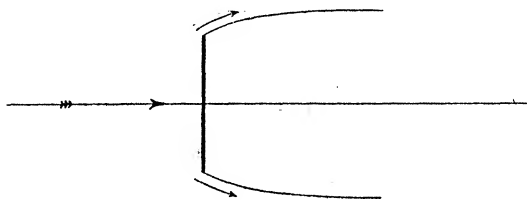


FIG. 5.—Discontinuous flow of a perfect fluid past a lamina at right angles to the stream.

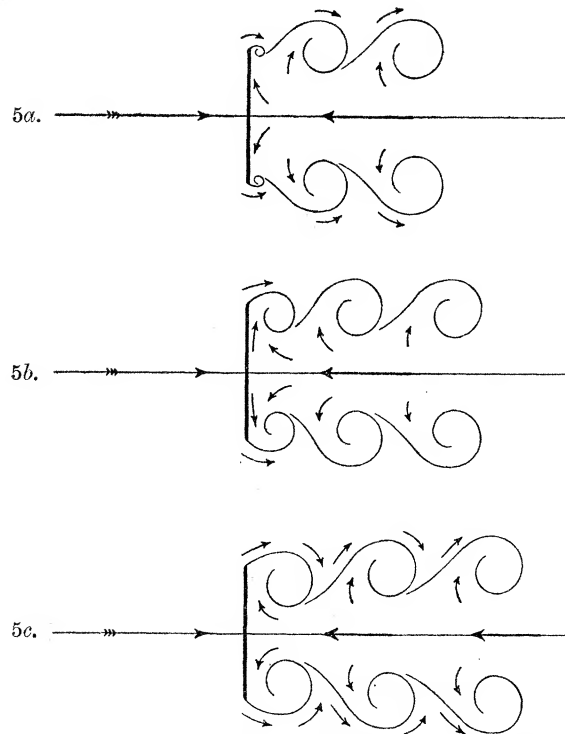
If the lamina were devoid of surface friction, and the liquid free from viscosity, this form of flow would be possible, though unstable.

In the actual case the fluid which is in contact with the up stream surface of the lamina is degraded and reaches the edge with a velocity insufficient to follow the theoretical curve. The result is that the fluid as it passes the edge is necessarily deflected inwards behind the down stream face of the lamina.

The subsequent motion cannot be followed by analysis. Observation, however, shows that the flow is not steady but of a periodic character, the actual period being determined by the velocity of the stream, the magnitude of lamina, and the angle between it and the general direction of flow.

Figs. 5*a*, 5*b*, 5*c* indicate the periodic structure of the wake in the case of a long lamina at right angles to the stream.

The inflow past the edge initiates a small eddy, about half of the fluid



FIGS. 5*a*, 5*b*, 5*c*.—Diagram illustrating the periodic formation of eddies in a viscous fluid under the same conditions as in Fig. 5.

involved being derived from the wake (or dead water behind the lamina) and half from the stream. The eddy continues to grow, and the conditions of continuity demand a current in the interior of the wake in the contrary direction to that of the outside stream to supply part of the fluid required for its formation. When the diameter of the eddy exceeds a certain fraction of the width of the lamina, the access of a sufficient supply from the central wake is impeded, and the full-grown eddy then leaves its position close to the lamina and travels down stream, while a fresh eddy is being formed in the same way at the edge. The general structure of the wake, therefore, consists not in a mass of dead water separated from the stream by a surface of discontinuity, but of a central current opposite to that of the stream bordered by a series of eddies at definite distances from one another.

It may be mentioned that the eddies just spoken of may be in the same state of growth at either edge, or be formed alternately (fig. 6). In the latter case the full-grown eddy occupies nearly the whole width of the lamina, and the wake consists of a trail of eddies alternately right-handed and left-handed.

How far the diameter of the full-grown eddy remains directly proportional to the width of the lamina it would be difficult to say. I have observed this proportionately up to a width of 20 cm., but would it be true when the width was as many kilometres? Under dynamically similar conditions it should be.

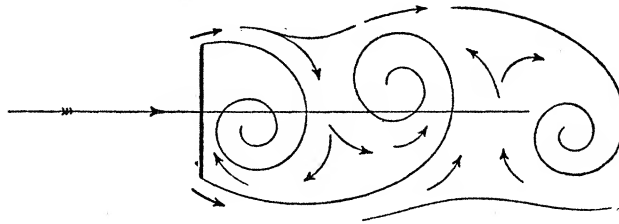


FIG. 6.—Diagram showing the alternate formation of right-handed and left-handed eddies in a viscous fluid.

(8) When the obstacle is not a lamina but a cylinder, the character of the wake is not greatly altered, and the eddies may be formed either simultaneously or alternately. In the alternate formation, the flow past the obstacle is more rapid round that side on which the eddy is growing than on the other, and the stream line pressure therefore less. Hence the force acting on the obstruction is not in the direction of the stream, but has a lateral component urging the obstruction towards that side.

(9) If the cylinder is a stretched string with a period of its own, and if the velocity of the stream is such that the period of eddy formation approaches the natural period of the string, it will be seen that the applied force is in the right phase to maintain a vibration when once started. This is the explanation of the action of the *Æolian harp* and many allied phenomena.

(10) The whistling of the wind is also a result of the periodic formation of eddies, but in this case the pitch of the note is in general dependent only on the velocity of the wind and the dimensions of the obstacle and not a coincidence of periods, and a sound will be produced whether the formation of the eddies is synchronous or alternate.

Many more examples might be given, but those already cited are sufficient to illustrate the far-reaching effects of the viscous degradation of potential, and of the principle to which Froude called attention, as explaining the difference in the behaviour of a viscous fluid, according to whether the motion is towards regions of increasing or diminishing pressure.